The second mid-term exam in the advanced econometrics (S3)
19.02.2008

ANSWER ALL QUESTIONS.

1a) The Gauss-Markov conditions for the linear multiple regression model ($\mathbf{Y} = \mathbf{X}\beta + \mathbf{u}$) are

$$\begin{aligned} &1. \quad E(\mathbf{u}|\mathbf{X}) = \mathbf{0_n}, \\ &2. \quad E(\mathbf{uu}'|\mathbf{X}) = \sigma_{\mathbf{u}}^2 \mathbf{I_n} \text{ and} \\ &3. \quad \mathbf{X} \text{ has full column rank.} \end{aligned} \tag{1}$$

Write down the Gauss-Markov theorem for multiple regression and explain shortly what it means. (3p)

b) In the GLS estimation we transform the original regression equation

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{u} \tag{2}$$

to

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{X}}\beta + \tilde{\mathbf{u}}, \tag{3}$$

where $\tilde{\mathbf{Y}} = \mathbf{FY}$, $\tilde{\mathbf{X}} = \mathbf{FX}$, $\tilde{\mathbf{u}} = \mathbf{Fu}$, $\mathbf{F'F} = \mathbf{\Omega^{-1}}$ and $E(\mathbf{uu}') = \mathbf{\Omega}$. Show that the error term $\tilde{\mathbf{u}}$ is homoscedastic. (4p)

c) What is the benefit of the GLS estimation in comparison with the OLS estimation and what does it have to do with the a)-part above? (2p)

2a) We have talked in the lectures about (at least) two ways of exploiting the property that the variable is correlated with its own past values and/or with the ones of the regressor. What are these? (2p)

b) Define the weak stationarity and the strong stationarity and describe both their properties and the differences. (4p)

c) Suppose we are interested in finding the most suitable number of lags p to be included into the $AR(p)$ model. The Bayes and Akaike information criteria are given as:

$$\begin{aligned} BIC(p) &= \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{\ln T}{T} \quad \text{and} && (4) \\ AIC(p) &= \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{2}{T}, && (5) \end{aligned}$$

where T is the number of observations, that is now 172 and $SSR(p)$ is the sum of squared residuals of the estimated $AR(p)$ model. The regression results for different models are given in the table

p	$SSR(p)/T$	$\ln(SSR(p)/T)$	$(p+1)\ln(T)/T$	$2(p+1)/T$
0	2.900	1.065	0.030	0.012
1	2.737	1.007	0.060	0.023
2	2.375	0.865	0.090	0.035
3	2.311	0.838	0.120	0.047
4	2.309	0.837	0.150	0.058

Which of the above models is the best according to the Bayes information criterion? How about the Akaike information criterion? How do the BIC and AIC differ from each other? Is this seen here? Justify your answers. (4p)

3a) We can write the heteroscedasticity- and autocorrelation consistent (HAC) estimator for the variance of the OLS estimator $\hat{\beta}_1$ (for the model $Y_t = \beta_0 + \beta_1 X_t + u_t$) as follows:

$$\tilde{\sigma}^2_{\hat{\beta}_1} = \hat{\sigma}^2_{\hat{\beta}_1} \hat{f}_T, \tag{6}$$

where $\hat{\sigma}^2_{\hat{\beta}_1}$ is the estimator of the variance of $\hat{\beta}_1$ without the serial correlation and $\hat{f}_T$ is the estimator for f_T, where

$$f_T = 1 + 2\sum_{j=1}^{T-1} \left(\frac{T-j}{T}\right)\rho_j, \tag{7}$$

where ρ_j stands for the j th autocorrelation. Describe a problem that appears in the estimation of the HAC standard errors. How can we circumvent this problem? (4p)

b) Define the two different types of exogeneity talked in the lectures. For what do we need the strong exogeneity? (3p)

4. Give an iterated two-period ahead forecast (as a function of parameter estimates $\hat{\beta}_0, \ldots, \hat{\beta}_3$ and the information up to period T) using the $AR(3)$ model

$$\hat{Y}_{T+2|T} = \hat{\beta}_0 + \hat{\beta}_1 \hat{Y}_{T+1|T} + \hat{\beta}_2 Y_T + \hat{\beta}_3 Y_{T-1}. \tag{8}$$

(4p)